01.08.2023 – Second Prize Winner Mr.Peeyush Pande's Solution

Prove : MR = PT

Construction : Join BP, PC

Proof:

In ΔRTC by Thales theorem

$\frac{TM}{MR} = \frac{TS}{SC}$

$\Longrightarrow$ MR = $\frac{SC \times TM}{ST}$ -------------------(1)

$\angle CBP = \angle CAP$ (same Chord)

In ΔADC

$\angle DAC = 90 - \angle C$

$\therefore \angle CBP = 90 \angle C = \angle CRP$ $(same\ chord)$

In ΔBFC,

$\angle BCF = 90 - \angle B$

$\angle BPR = 90 - \angle B$ $(same\ chord)$

$\angle BSM = \angle BCR = 90 - \angle B$ (Corresponding ∠, SM ∥ CR)

∴In ΔBTP, ΔMTS

$\angle TBP = \angle TMS = 90 - \angle C$

$\angle TSM = \angle TBP = 90 - \angle B$

$\therefore \Delta BTP \sim \Delta MTS$ (AA)

$\frac{BT}{TP} = \frac{MT}{TS}$ ------------------ (2)

In ΔSPC

$\angle SPC = \angle QPC$

$= \angle QBC$

= 90-$\angle ECB$

= 90 - $\angle C$ (ECB right Δ)

$\angle SCP = \angle BCP$

= $\angle BQP$

$= \angle BAP$

=90 - $\angle ABD$ (ABD right Δ)

= 90- $\angle B$

∴ In Δ BTP, ΔPSC

$\angle TBP = \angle SPC$

$\angle TPB = \angle SCP$

∴ ΔBTP ~ ΔPSC

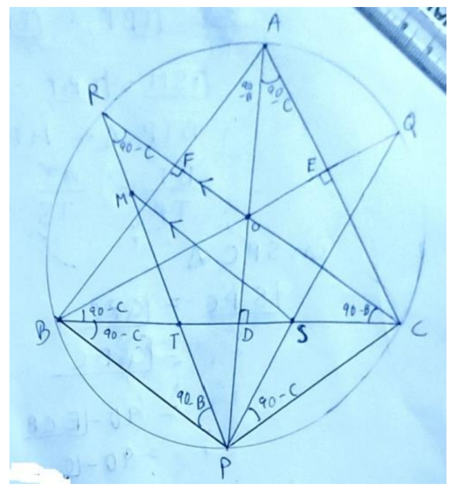
$\frac{BT}{TP} = \frac{PS}{SC}$ ------------------- (3)

By (2) & (3)

$\therefore \frac{MT}{TS} = \frac{PS}{SC}$

$\therefore by\ (1)$

MR = $\frac{SC \times TM}{ST}$



= SC $\times \frac{TM}{TS}$

= SC $\times \frac{PS}{SC}$

= PS

$\therefore MR = PS$

$\angle APQ = \angle ABQ$ (same chord)

= 90- $\angle A$ (In ABE right Δ)

$\angle APR = \angle ACT$ (same chord)

= 90- $\angle A$ (In ABE right Δ)

$\therefore \angle APQ = \angle ACR$

=90-$\angle A$

$\therefore \angle APR = \angle ACR$

In ΔAPS altitude PD = angle bisection

$\therefore$ It is isosceles

SP = PT

$\therefore MR = PT$ -------- Hence Proved

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